MATH 2230BC Tutorial note 7

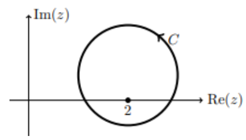
1 Tutorial questions

1. Compute

$$\int_C \frac{e^{z^2}}{z-2} dz$$

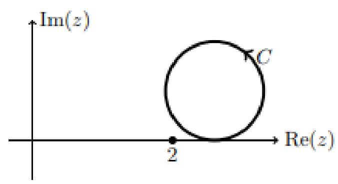
where C is the curve shown

Graph



(a)

Graph



(b)

Solution

(a) Let $f(z) = e^{z^2}$ is entire. Since C is a simple closed curve (counterclockwise) and $z = 2$ is inside C. By Cauchy's integral formula, we have

$$\int_C \frac{e^{z^2}}{z-2} dz = 2\pi i f(2) = 2\pi i e^4.$$

(b) Since $\frac{e^{z^2}}{z-2}$ is analytic on and inside C. Then

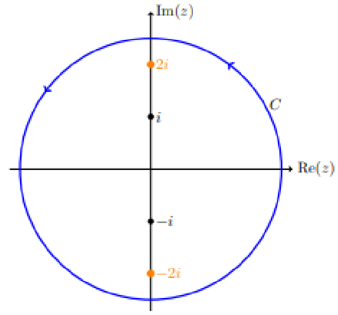
$$\int_C \frac{e^{z^2}}{z-2} dz = 0.$$

2. Compute

$$\int_C \frac{z}{z^2+4} dz$$

where C is the curve shown

Graph



Solution We write

$$\frac{z}{z^2+4} = \frac{z}{(z-2i)(z+2i)}.$$

Let

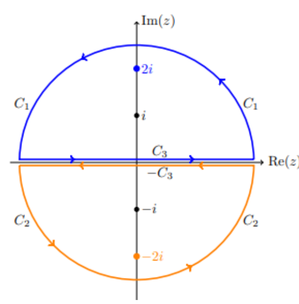
$$f_1(z) = \frac{z}{z+2i}, \qquad f_2(z) = \frac{z}{z-2i}.$$

Then

$$\frac{z}{z^2+4} = \frac{f_1(z)}{z-2i} = \frac{f_2(z)}{z+2i}.$$

Split the original curve C into 2 pieces that each surround just one singularity

Graph



The integral becomes

$$\int_C \frac{z}{z^2+4}dz = \int_{C_1+C_3-C_3+C_2} \frac{z}{z^2+4}dz = \int_{C_1+C_3} \frac{z}{z^2+4}dz + \int_{-C_3+C_2} \frac{z}{z^2+4}dz$$

Note that the function $f_1(z)$ is analytic inside the C_1+C_3 and $f_2(z)$ is analytic inside the $-C_3+C_2$. We replace $\frac{z}{z^2+4}$ by $\frac{f_1(z)}{z-2i}$ and $\frac{f_2(z)}{z+2i}$ in the intgral along C_1+C_3 and $-C_3+C_2$ respectively,

$$\begin{aligned}\int_C \frac{z}{z^2+4}dz &= \int_{C_1+C_3} \frac{f_1(z)}{z-2i}dz + \int_{-C_3+C_2} \frac{f_2(z)}{z+2i}dz \\ &= 2\pi i f_1(2i) + 2\pi i f_2(-2i) \\ &= 2\pi i\end{aligned}$$

2 Homework problems

Please refer solution for HW7.